<table>
<tr><th></th><th></th><th>DL syntax</th><th>Protégé</th><th>Python + Owlready2</th><th>First-order logic</th><th>Semantics in set formula</th></tr>
<tr><td rowspan="2">Const.</td><td>Top</td><td>$\top$</td><td>Thing</td><td>Thing</td><td>$\top$, such as $\forall x, \top(x) = true$</td><td>$\Delta$</td></tr>
<tr><td>Bottom</td><td>$\bot$</td><td>Nothing</td><td>Nothing</td><td>$\bot$, such as $\forall x, \bot(x) = false$</td><td>$\emptyset$</td></tr>
<tr><td rowspan="5">Axioms</td><td>Subsumption</td><td>$A \sqsubseteq B$</td><td>A subclass of B</td><td>class A(B): ... (assertion)
A.is_a.append(B) (assertion)
issubclass(A, B) (test)</td><td>$\forall x, A(x) \rightarrow B(x)$</td><td>$f(A) \subseteq f(B)$</td></tr>
<tr><td></td><td>$R \sqsubseteq S$</td><td>R subproperty of S</td><td>(same as above)</td><td>$\forall x \forall y, R(x,y) \rightarrow S(x,y)$</td><td>$f(R) \subseteq f(S)$</td></tr>
<tr><td>Equivalence</td><td>$A \equiv B$</td><td>A equivalent to B</td><td>A.equivalent_to.append(B) (as.)
B in A.equivalent_to (test)</td><td>$\forall x, A(x) \leftrightarrow B(x)$</td><td>$f(A) = f(B)$</td></tr>
<tr><td>Instanciation</td><td>$A(i)$</td><td>i type A</td><td>i = A() (assertion)
i.is_instance_of.append(A)
isinstance(i, A) (test)</td><td>$A(i)$</td><td>$f(i) \in f(A)$</td></tr>
<tr><td>Relations</td><td>$R(i,j)$</td><td>i object property assertion j
i data property assertion j</td><td>i.R = j (R is functional)
i.R.append(j) (otherwise)</td><td>$R(i,j)$</td><td>$(f(i), f(j)) \in f(R)$</td></tr>
<tr><td rowspan="14">Semantic connectors</td><td>Complement</td><td>$\neg A$</td><td>not A</td><td>Not(A)</td><td>$\neg A(x)$</td><td>$\Delta \backslash f(A)$</td></tr>
<tr><td>Intersection</td><td>$A \sqcap B$</td><td>A and B</td><td>A & B (or) And([A, B,...])</td><td>$A(x) \wedge B(x)$</td><td>$f(A) \cap f(B)$</td></tr>
<tr><td>Union</td><td>$A \sqcup B$</td><td>A or B</td><td>A | B (or) Or([A, B,...])</td><td>$A(x) \vee B(x)$</td><td>$f(A) \cup f(B)$</td></tr>
<tr><td>Extension</td><td>$i, j, ...$</td><td>{i, j,...}</td><td>OneOf([i, j,...])</td><td>$x \in \{i, j, ...\}$</td><td>$\{f(i), f(j), ...\}$</td></tr>
<tr><td>Inverse</td><td>R^-</td><td>inverse of R</td><td>Inverse(R) (construct)
S.inverse = R (assertion)</td><td>$\forall i \forall j, S(i,j) = R(j,i)$</td><td>$\{(a,b) \mid (b,a) \in f(R)\}$</td></tr>
<tr><td>Transitive closure</td><td>R^+</td><td>-</td><td>-</td><td></td><td>$\cup_{i \geq 1} (f(R))^i$</td></tr>
<tr><td>Composition</td><td>$R \circ S$</td><td>R o S</td><td>PropertyChain([R, S])</td><td></td><td>$\{(a,c) \in \Delta \times \Delta \mid \exists b, (a,b) \in f(R) \wedge (b,c) \in f(S)\}$</td></tr>
<tr><td>Existential quantifier</td><td>$\exists R.B$</td><td>R some B</td><td>R.some(B)</td><td>$\exists y, R(x,y) \wedge B(y)$</td><td>$\{a \in \Delta \mid \exists b, (a,b) \in f(R) \wedge b \in f(B)\}$</td></tr>
<tr><td>Universal quantifier</td><td>$\forall R.B$</td><td>R only B</td><td>R.only(B)</td><td>$\forall y, R(x,y) \rightarrow B(y)$</td><td>$\{a \in \Delta \mid \forall b, (a,b) \in f(R) \rightarrow b \in f(B)\}$</td></tr>
<tr><td>Number restrictions</td><td>$= 2R.B$</td><td>R exactly 2 B</td><td>R.exactly(2, B)</td><td>$|\{y \mid R(x,y) \wedge B(y)\}| = 2$</td><td>$\{a \in \Delta \mid |\{b \mid (a,b) \in f(R) \wedge b \in f(B)\}| = 2\}$</td></tr>
<tr><td></td><td>$\leq 2R.B$</td><td>R max 2 B</td><td>R.max(2, B)</td><td>$|\{y \mid R(x,y) \wedge B(y)\}| \leq 2$</td><td>$\{a \in \Delta \mid |\{b \mid (a,b) \in f(R) \wedge b \in f(B)\}| \leq 2\}$</td></tr>
<tr><td></td><td>$\geq 2R.B$</td><td>R min 2 B</td><td>R.min(2, B)</td><td>$|\{y \mid R(x,y) \wedge B(y)\}| \geq 2$</td><td>$\{a \in \Delta \mid |\{b \mid (a,b) \in f(R) \wedge b \in f(B)\}| \geq 2\}$</td></tr>
<tr><td>Role filler</td><td>$\exists R.\{j\}$</td><td>R value j</td><td>R.value(j)</td><td>$R(x,j)$</td><td>$\{a \in \Delta \mid (a, f(j)) \in f(R)\}$</td></tr>
<tr><td rowspan="5">Decomposable</td><td>Disjoint</td><td>$A \sqcap B \sqsubseteq \bot$</td><td>A disjoint with B</td><td>AllDisjoint([A, B])</td><td>$\forall x, \neg(A(x) \wedge B(x))$</td><td>$f(A) \cap f(B) = \emptyset$</td></tr>
<tr><td>Property domain</td><td>$\exists R.\top \sqsubseteq A$</td><td>R domain A</td><td>R.domain = [A]</td><td>$\forall x, (\exists y, R(x,y)) \rightarrow A(x)$</td><td>$f(R) \subseteq \{(a,b) \mid a \in f(A)\}$</td></tr>
<tr><td>Property range</td><td>$\top \sqsubseteq \forall R.B$</td><td>R range B</td><td>R.range = [B]</td><td>$\forall x \forall y, R(x,y) \rightarrow B(y)$</td><td>$f(R) \subseteq \{(a,b) \mid b \in f(B)\}$</td></tr>
<tr><td>Role filler as class property</td><td>$A \sqsubseteq \exists R.\{j\}$
$\wedge (\exists R^-.A)(j)$</td><td>-</td><td>A.R = j (R is functional)
A.R.append(j) (otherwise)</td><td>-</td><td>-</td></tr>
<tr><td>Local closed world</td><td>-</td><td>-</td><td>close_world(A)</td><td>-</td><td>-</td></tr>
</table>

The great table of Description Logics and formal ontology notations (logics, programming and semantics) —